

Probability &
Statistics

Sections 2.5 and 5.1

Introduction to Normal Distributions and the Standard Distribution

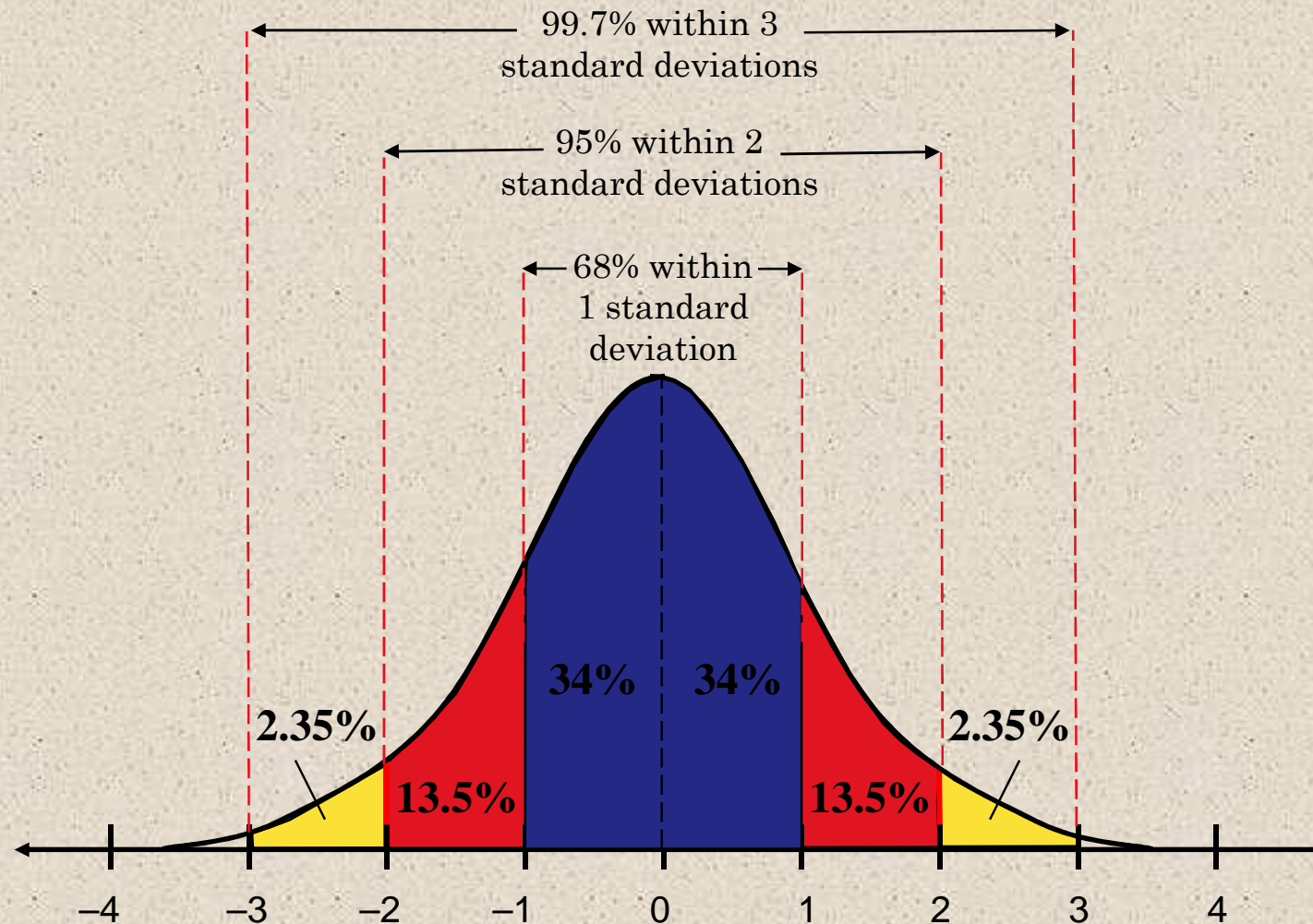
Empirical Rule (68-95-99.7%)

Empirical Rule

For data with a (symmetric) bell-shaped distribution, the standard deviation has the following characteristics.

1. About 68% of the data lie within one standard deviation of the mean.
2. About 95% of the data lie within two standard deviations of the mean.
3. About 99.7% of the data lie within three standard deviation of the mean.

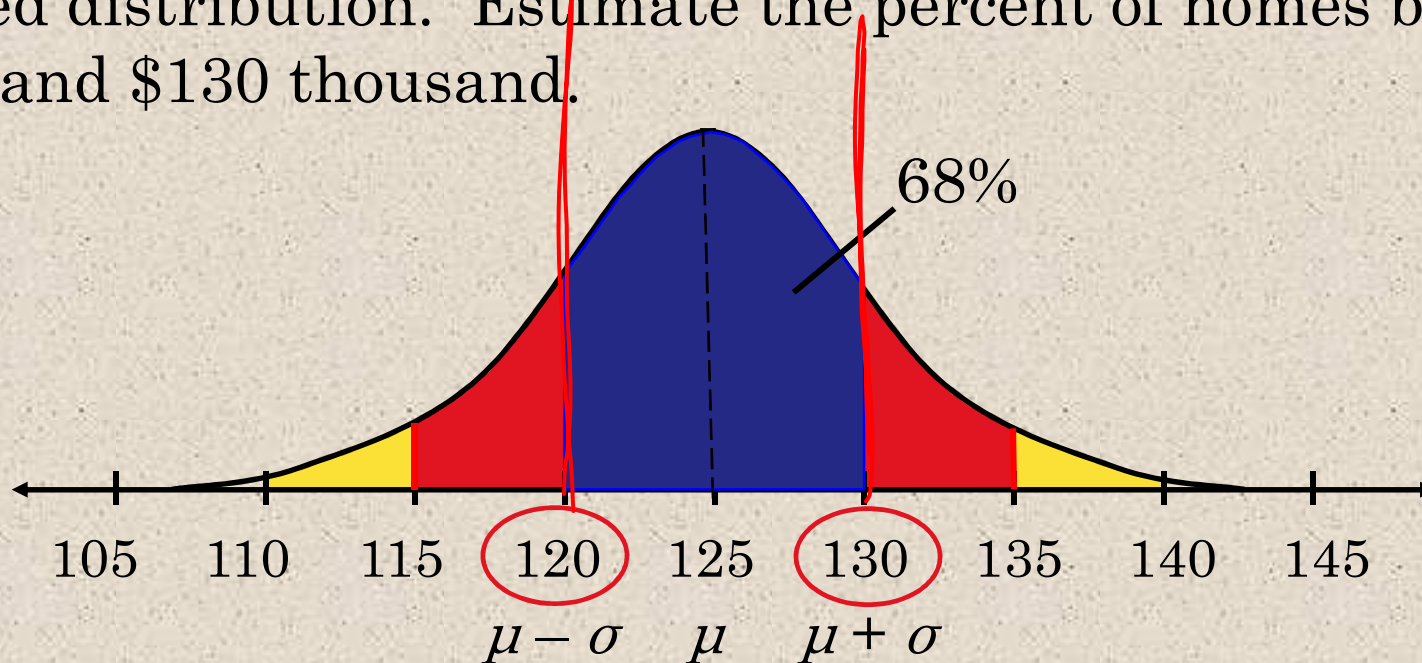
Empirical Rule (68-95-99.7%)



Using the Empirical Rule

Example:

The mean value of homes on a street is \$125 thousand with a standard deviation of \$5 thousand. The data set has a bell shaped distribution. Estimate the percent of homes between \$120 and \$130 thousand.



68% of the houses have a value between \$120 and \$130 thousand.

Using the Empirical Rule

Suppose the class average on a test is 87% and the standard deviation of the class is 3. Find:

1. The percentage of students who scored $> 90\%$.

15.85%

2. The percentage of students who scored between 84% and 90%.

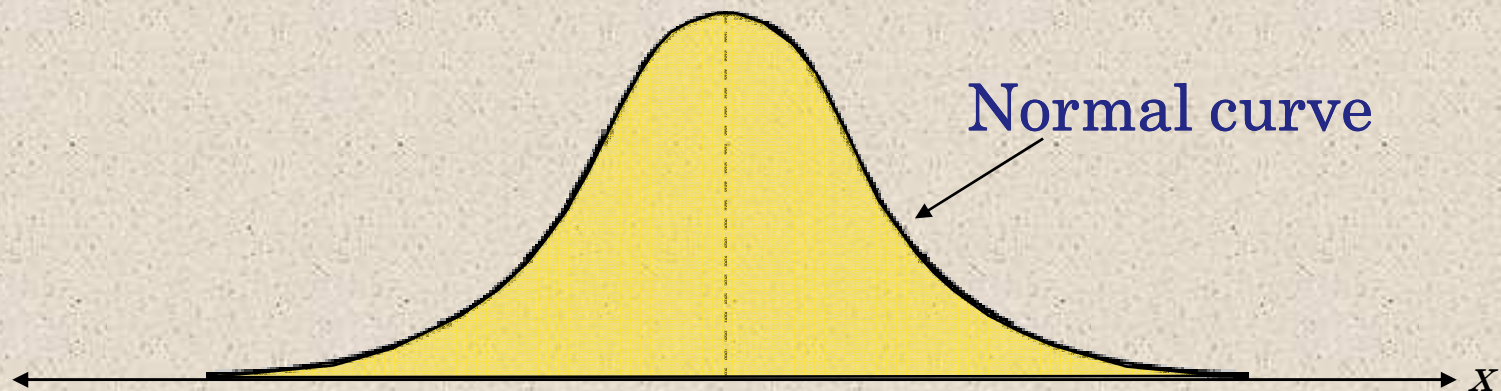
68%

3. The percentage of students who scored $< 84\%$.

15.85%

Properties of Normal Distributions

The most important probability distribution in statistics is the **normal distribution**.



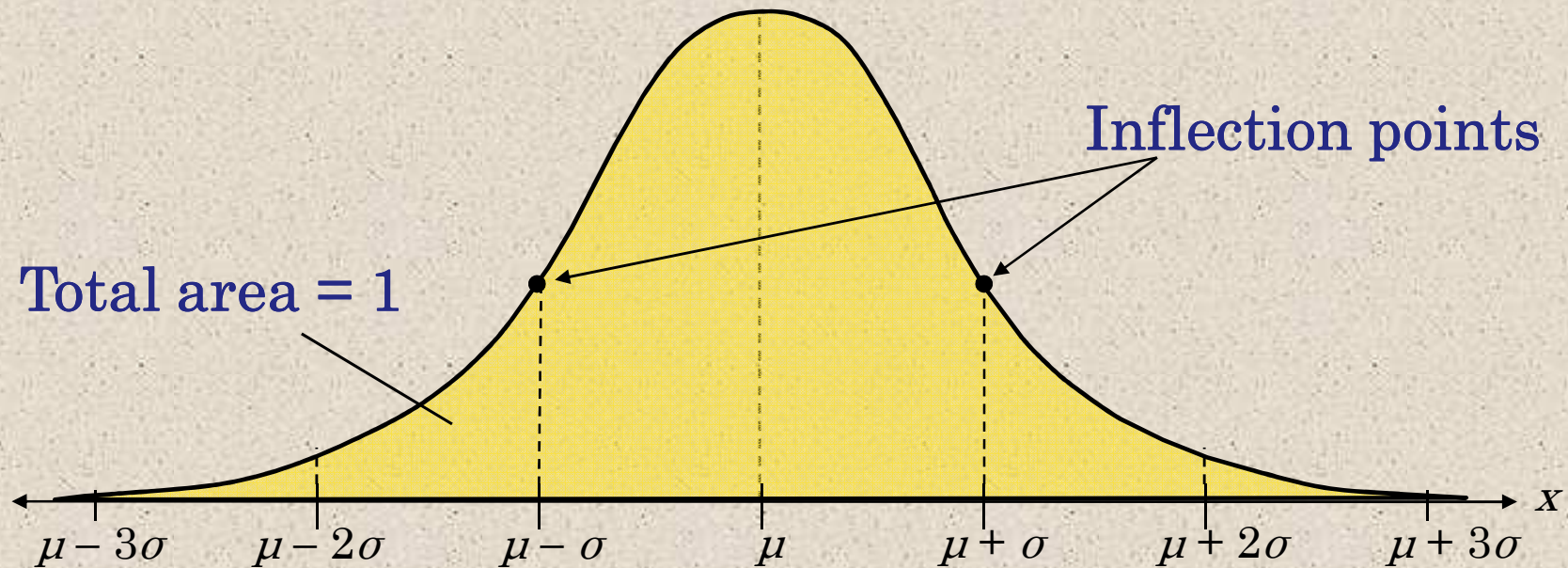
A normal distribution is a continuous probability distribution for a random variable, x . The graph of a normal distribution is called the **normal curve**.

Properties of Normal Distributions

Properties of a Normal Distribution

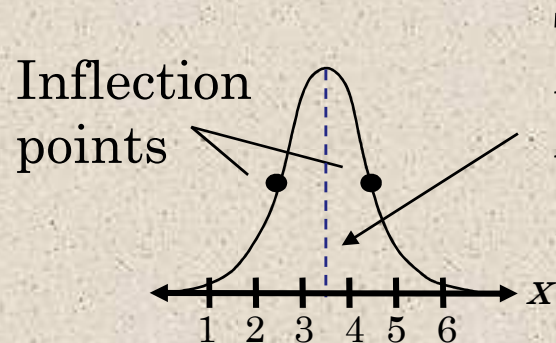
1. The mean, median, and mode are equal.
2. The normal curve is bell-shaped and symmetric about the mean.
3. The total area under the curve is equal to one.
4. The normal curve approaches, but never touches the x -axis as it extends farther and farther away from the mean.
5. Between $\mu - \sigma$ and $\mu + \sigma$ (in the center of the curve), the graph curves downward. The graph curves upward to the left of $\mu - \sigma$ and to the right of $\mu + \sigma$. The points at which the curve changes from curving upward to curving downward are called the *inflection points*.

Properties of Normal Distributions



Means and Standard Deviations

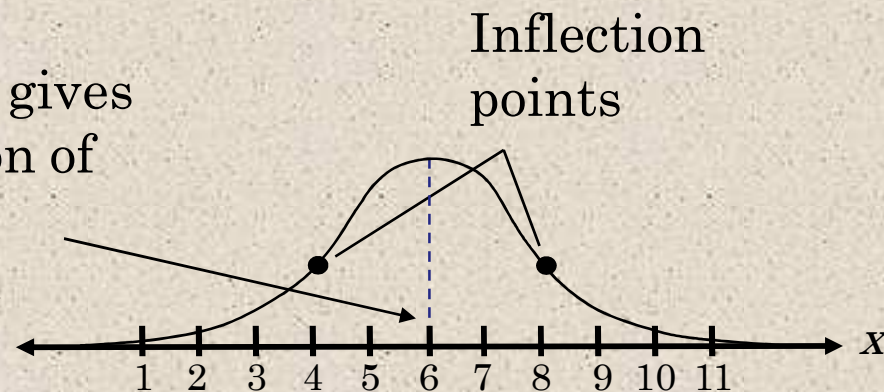
A normal distribution can have any mean and any positive standard deviation.



The mean gives the location of the line of symmetry.

Mean: $\mu = 3.5$

Standard deviation: $\sigma \approx 1.3$



Mean: $\mu = 6$

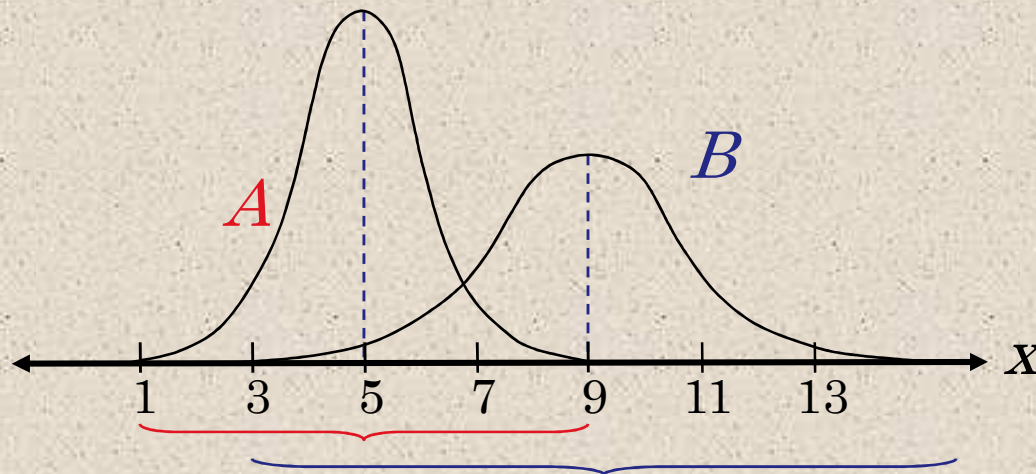
Standard deviation: $\sigma \approx 1.9$

The standard deviation describes the spread of the data.

Means and Standard Deviations

Example:

1. Which curve has the greater mean?
2. Which curve has the greater standard deviation?



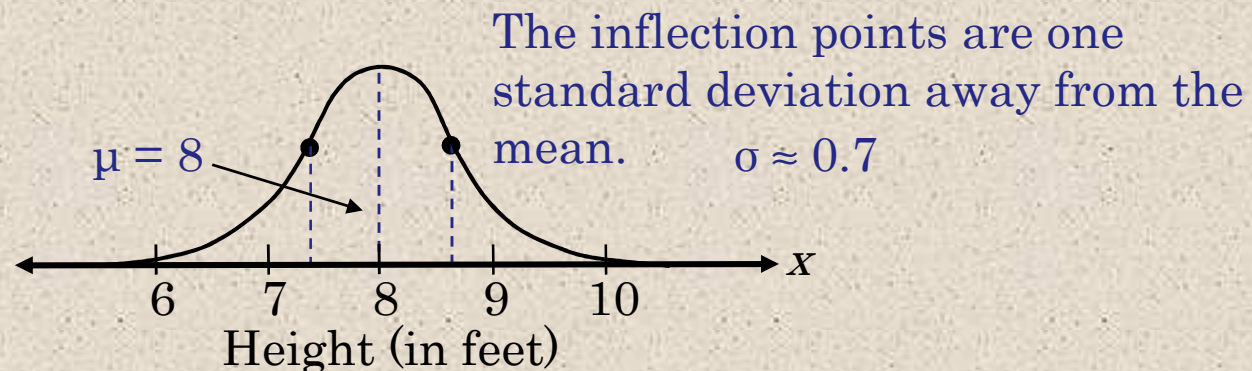
The line of symmetry of curve A occurs at $x = 5$. The line of symmetry of curve B occurs at $x = 9$. Curve B has the greater mean.

Curve B is more spread out than curve A , so curve B has the greater standard deviation.

Interpreting Graphs

Example:

The heights of fully grown magnolia bushes are normally distributed. The curve represents the distribution. What is the mean height of a fully grown magnolia bush? Estimate the standard deviation.



The heights of the magnolia bushes are normally distributed with a mean height of about 8 feet and a standard deviation of about 0.7 feet.